s with the nuclei becomes in
e substance may be regarded
erfect Fermi electron gas,
of dK/dp in the nonrelativi
lativistic approximations is
tively. It should be noted th
ply only at extremely high pr
to the same source, the va
atm $\gg p \gg 5 \times 10^{4}Z^{10/3}$ at
average atomic number of t
as the value 4/3 is for $p \gg 10$
n ($Z = 11$) the inequality f
ecomes $10^{17} \gg p \gg 1.5 \times 10$
is far above the range of an
a and probably even above th
trapolations are needed! Th
pressure found in compiling th
ons is a shock wave point a
for aluminum oxide. *Birch*
ated the pressure at the cente
be of the order of 3.4 Mb. In
sidered normal for dK/dp to
n a monotone fashion as th
s. Equation 2 provides th
or, but the leveling off o
a few per cent of the value m
ssures p of the order of $10aK$
able values of a, is very low
^{12} atm. Therefore, in order to
cted behavior over the pre
he extrapolation is desired, it
t the best m to use in equa
ubstantially larger than 5/3
ensitive point, however, sinc
or C) remains undetermined
ue of the second derivative
3, can still be adjusted by

e first two pressure deriva
nodulus at $P = 0$ and th
e first derivative as $P \rightarrow$
o the corresponding valu
quation by using the sam
a in the two equations a
', or, equivalently, C
similar match to the Bir
$= 4$ [*Birch*, 1938, 1952
and $C = -35/9$, where
tch requires $m = 3$ a
$-143/9$.
s to be presented here, w
$= 5/3$ when $C < 0$, an

For C and K_0' both positive we used

$$m - K_0' = \frac{C}{2[K_0' - (2C)^{1/2}]} \qquad (4)$$

This expression was obtained as an approximation to the smallest value of m that allowed K to fall to zero on $-a < P < 0$, assuming $C \ll K_0'^2$. The condition that K drop continuously to zero on $P < 0$ may be regarded as an 'instability' condition. It is satisfied automatically whenever $K_0' > 0$ and $C < 0$. It is not considered essential for the purpose of extrapolating on $P > 0$, but in the absence of any other guidance, it seemed to be a reasonable criterion for relating the two adjustable parameters, say m and C, when K_0' and C are both positive. The idea that m should be near the smallest value that allows this instability follows from the feeling that the condition $m > K_0'$, needed to avoid a singularity on $P > 0$ in this case, is likely to give an m that is already too large to be a correct limiting value of dK/dp as $p \rightarrow \infty$.

When $K_0' < 0$, as for vitreous silica, many formulas including those of Murnaghan and Keane necessarily predict an instability ($K \leq 0$ on $P > 0$. Although this may not be a great catastrophe, and could even be represented as advantageous (because an actual material with $K_0' < 0$ could be presumed to undergo a phase transition, through which the extrapolation should not be continued analytically), it is interesting to note that the present formula allows such an instability to be avoided by choosing a sufficiently high positive value for C. This is illustrated in Figure 1, which shows K/K_0 versus P for three different values of C with $K_0' = -6.5$ and $m = 1$. The value $K_0' = -6.5$ applies to vitreous silica [McSkimin as cited by *Anderson*, 1961].

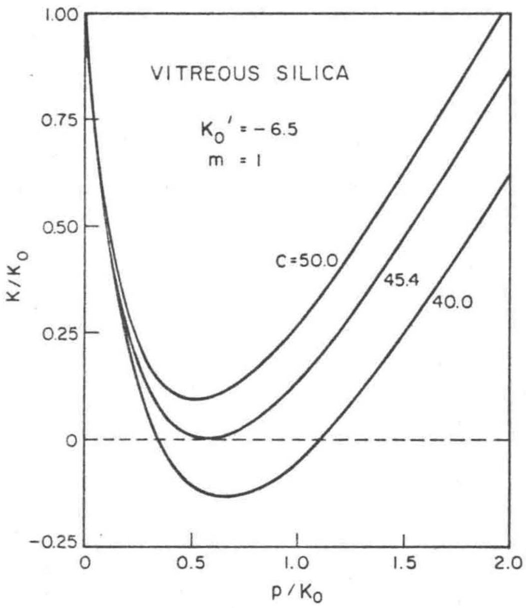

Fig. 1. Determining the value of C that ensures reasonable behavior of K on $p > 0$ for the anomalous case, vitreous silica ($K_0' < 0$) (see text).

Compression Equation

The next task is to relate the volume v to the pressure, subject to equation 2 and the definition of the bulk modulus

$$K = -v\, dp/dv \qquad (5)$$

Let $V = v/v_0$. Then

$$K/K_0 = -V\, dP/dV \qquad (6)$$

As an abbreviation in equation 2, let $A = a(K_0' - m)$. Then the integral of equation 2 is

$$\frac{K}{K_0} = -V\frac{dP}{dV} = 1 + A + mP - \frac{aA}{P + a} \qquad (7)$$

where the constant of integration has been determined to make $K = K_0$ at $P = 0$. From (7)

$$V = \exp\left[-\int \frac{dP}{\left(1 + A + mP - \dfrac{aA}{P + a}\right)}\right] \qquad (8)$$

The evaluation of the integral in the expression above, subject to $V = 1$ when $P = 0$ (Given in Appendix B), gives us

$$V = \left\{\left[\frac{a}{mP^2 + (1 + A + am)P + a}\right] \cdot \left[\frac{4am + 2mP[(q)^{1/2} + (1 + A + am)]}{4am - 2mP[(q)^{1/2} - (1 + A + am)]}\right]^{(1+A-am)/(q)^{1/2}}\right\}^{1/2m} \qquad (9)$$